Primordial Density Perturbations in EiBI Inflation

Inyong Cho
(SeoulTech)

Joint Winter Conference on Particle Physics, String and Cosmology
High1 Resort, Korea, Jan. 25-31, 2015

- **Inflation:**
  Phys. Rev. Lett. 111, 071301 (2013)
  I.C., Hyeong-Chan Kim (KNUT) & Taeyoon Moon (Inje Univ.)
- **Tensor Perturbation:**
  PRD 90, 024063 (2014)
  I.C., Hyeong-Chan Kim (KNUT)
- **Scalar Perturbation:**
  EPJC 74:3155 (2014) I.C., Naveen K. Singh
  arXiv:1412.6344 [gr-qc] I.C., Naveen K. Singh

Outline

1. Eddington-inspired Born-Infeld gravity
2. Inflation in EiBI gravity **('13 IC, KIM, Moon)**
3. Scalar Perturbation in EiBI Inflation **('14 IC, Singh)**
4. Tensor Perturbation in EiBI Inflation **('14 IC, KIM)**
5. Tensor-to-Scalar Raito
6. Conclusions

$8\pi G = 1$

Eddington-inspired Born-Infeld Gravity

$$\Lambda = \frac{\lambda - 1}{\kappa}$$

$$\boxed{S_{\text{EiBI}} = \frac{1}{\kappa}\int d^4x\Big[\sqrt{-|g_{\mu\nu}+\kappa R_{\mu\nu}(\Gamma)|} - \lambda\sqrt{-|g_{\mu\nu}|}\,\Big] + S_M(g,\Phi)}$$

(Vollick 2004, Banados-Ferreira 2010)

:- $\boxed{g_{\mu\nu} \text{ and } \Gamma^{\mu}_{\alpha\beta}}$: independent → Palatini Formalism

:- Matter is in usual way (Not in sqrt)

:- equivalent to bi-metric theory

EOM1: $\boxed{\frac{\sqrt{-|q|}}{\sqrt{-|g|}}\, q^{\mu\nu} = \lambda g^{\mu\nu} - \kappa T^{\mu\nu}}$: Relation b/t g and q via T

EOM2: $\boxed{q_{\mu\nu} = g_{\mu\nu} + \kappa R_{\mu\nu}}$:- "Auxiliary Metric"

:- Dynamical Equation

EOM for matter: $\boxed{\nabla^{g}_{\mu} T^{\mu\nu} = 0}$: Energy-Momentum Conservation

→ Matter plays in the background metric

(MERIT 1) One parameter (κ) theory

(MERIT 2) EiBI in vacuum or with only CC is the same with GR

⇨ **Schwartzchild-(Anti) de Sitter BH**

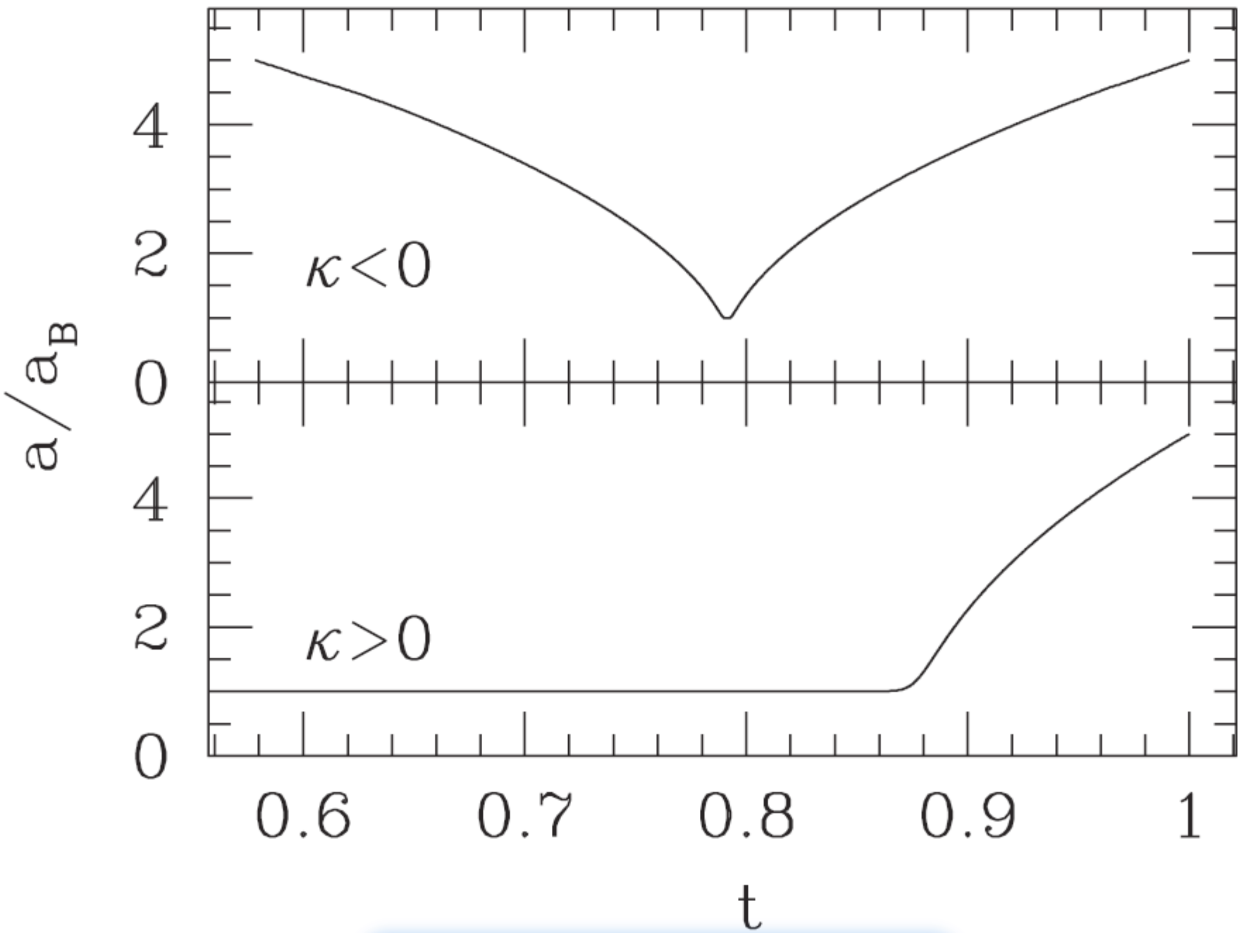
(3) Poisson Equation

$$\boxed{\nabla^2\Phi = -\frac{1}{2}\rho - \frac{1}{4}\kappa\nabla^2\rho.}$$

Implies repulsive nature of gravity

(4) Perfect Fluid → Not Big Bang Singularity

Banados & Ferreira for w=1/3 (2010)
IC, Kim and Moon for all w (2012)



Bouncing Universe

Non-Singular Universe

$$a(t) \approx a_B + Ae^{H_0 t}.$$

:- t → -infty : origin of Universe
:- finite size
:- Non-Singular Universe

Chaotic Inflation in GR

Action & Metric

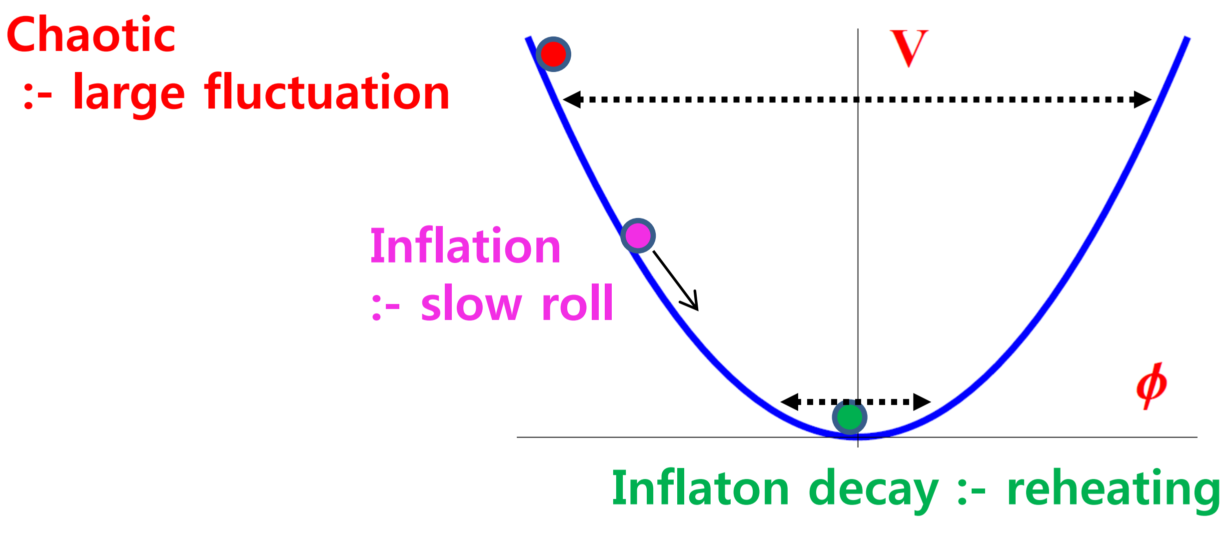
$$S_M = \int d^4x \sqrt{-|g_{\mu\nu}|} \left[-\frac{1}{2} g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi - V(\phi) \right], \qquad V(\phi) = \frac{m^2}{2}\phi^2, \qquad \boxed{8\pi G = 1}$$

$$g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t) d\mathbf{x}^2.$$

Field Equations & Slow-Roll Conditions

$$H^2 = \left(\frac{\dot a}{a}\right) = \frac{1}{3}\rho = \frac{1}{3}\left(\cancel{\frac{1}{2}\dot\phi^2} + V\right)$$: 1st slow-roll condition

$$\cancel{\ddot\phi} + 3H\dot\phi + V'(\phi) = 0$$: 2nd slow-roll condition



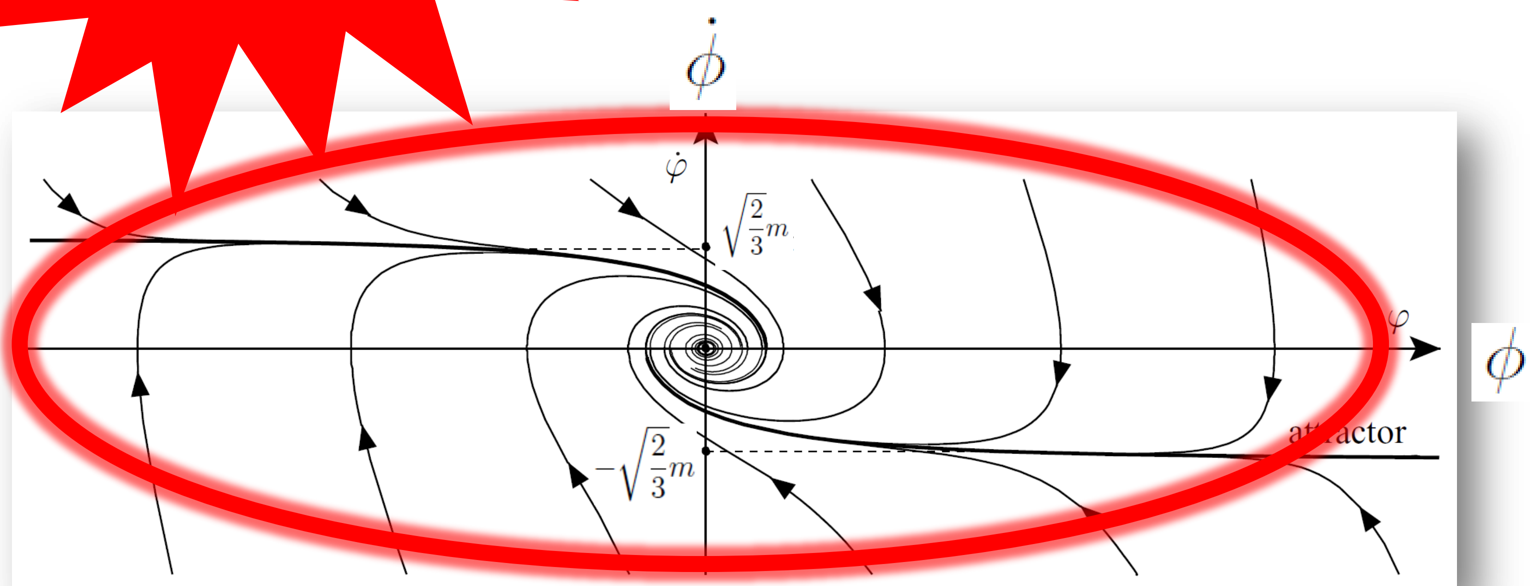
Attractor Solution for Slow-Roll Inflation

$$\phi(t) \approx \phi_i + \sqrt{2/3}mt, \qquad a(t) \approx a_i\, e^{\frac{1}{4}[\phi_i^2 - \phi^2(t)]},$$

$$\boxed{H^2 = \frac{1}{3}\rho}$$: curvature scale

$$\boxed{\rho = K + V > M_P^2}$$

Quantum Gravity Regime



EiBI Inflation

Metric

$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)\, d\mathbf{x}^2,$$
$$q_{\mu\nu}dx^\mu dx^\nu = -X^2(t)\, dt^2 + Y^2(t)\, d\mathbf{x}^2.$$

EOM1

$$X = (\lambda - \kappa p)^{3/4}(\lambda + \kappa\rho)^{-1/4}, \qquad Y = [(\lambda + \kappa\rho)(\lambda - \kappa p)]^{1/4}\, a,$$

Puts an upper limit on Pressure

$$\rho = \dot{\phi}^2/2 + V \text{ and } p = \dot{\phi}^2/2 - V$$

EOM2 : Friedmann Eq.

$$H \equiv \frac{\dot{a}}{a} = \frac{1}{(\bar{\lambda} + V)^2 + \dot{\phi}^4/2}\left\{-\frac{1}{2}\left(\bar{\lambda} + V + \frac{\dot{\phi}^2}{2}\right)V'(\phi)\dot{\phi} \pm \frac{1}{\sqrt{3}}\left(\bar{\lambda} + V - \frac{\dot{\phi}^2}{2}\right) \times \left[\left(\bar{\lambda} + V + \frac{\dot{\phi}^2}{2}\right)^{3/2}\left(\bar{\lambda} + V - \frac{\dot{\phi}^2}{2}\right)^{3/2} - \frac{1}{\kappa}\left(\bar{\lambda} + V + \frac{\dot{\phi}^2}{2}\right)\left(\bar{\lambda} + V - \dot{\phi}^2\right)\right]^{1/2}\right\}$$

$$\bar{\lambda} \equiv \lambda/\kappa$$

Matter-Field Eq.

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0.$$

Slow-Roll Evolution at ATTRactor

Already in pretty Low-energy Regime

Same for GR and EiBI

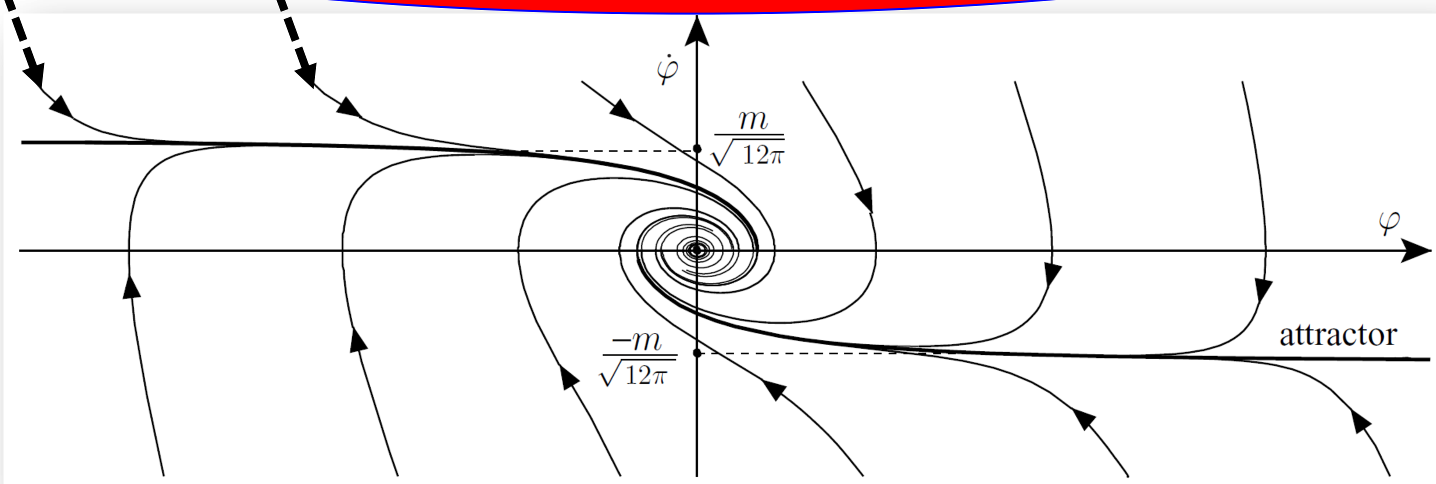
In EiBI gravity, there exists an upper limit in $\dot{\phi}^2$

→ Maximal Pressure Condition (MPC)

$$\boxed{\frac{1}{2}\dot{\phi}^2 - V(\phi) - \bar{\lambda} = 0,}$$

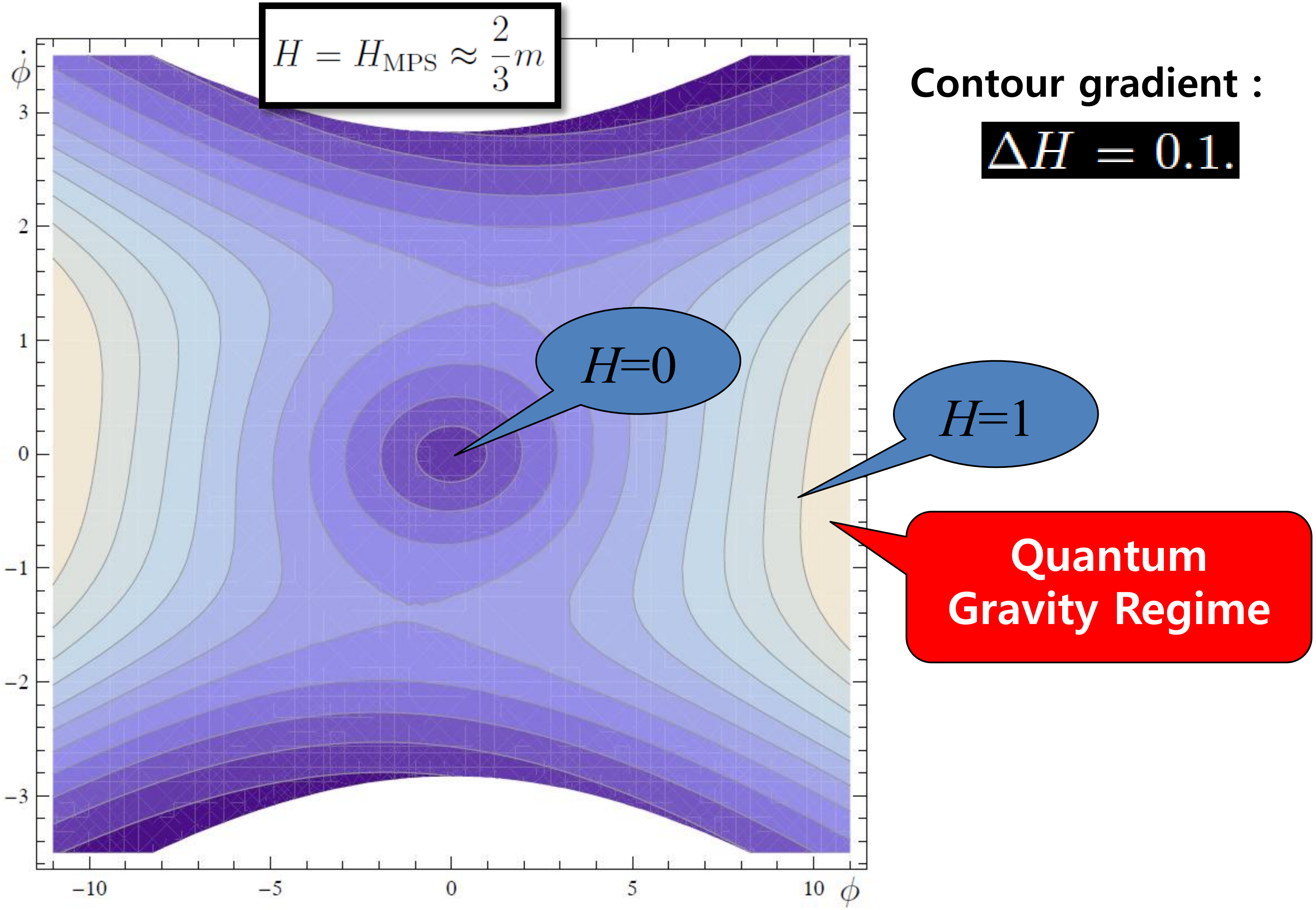
pressure

Forbidden



Curvature

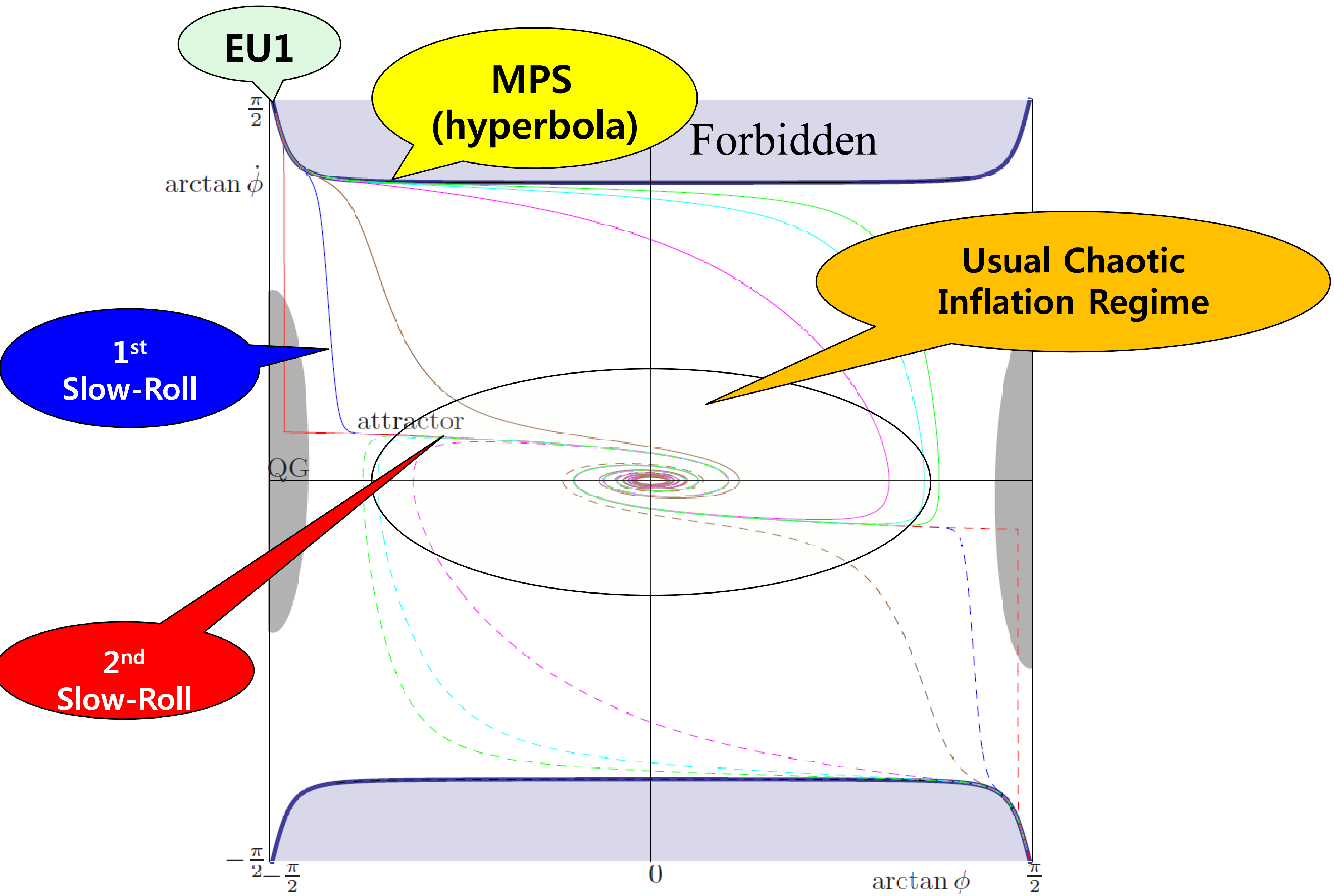
H in Phase Space $(\phi, \dot{\phi})$ for $m = 1/4,\ \kappa = 1/4,$ and $\lambda = 1.$



Contour gradient :

$\Delta H = 0.1.$

Evolution Path of Universe



Scalar Perturbation

Action

For λ=1, equivalent to bi-metric theory:

$$S[g,q,\varphi] = \frac{1}{2}\int d^4x\sqrt{-|q_{\mu\nu}|}\left[R(q) - \frac{2}{\kappa}\right] + \frac{1}{2\kappa}\int d^4x\left(\sqrt{-|q_{\mu\nu}|}q^{\alpha\beta}g_{\alpha\beta} - 2\sqrt{-|g_{\mu\nu}|}\right) + S_{\mathrm{M}}[g,\varphi],$$

Metric Perturbation

$$ds_q^2 = b^2\left\{-\frac{1+2\phi_1}{\boxed{z}}d\eta^2 + 2\frac{B_{1,i}}{\sqrt{z}}d\eta dx^i + \left[(1-2\psi_1)\delta_{ij} + 2E_{1,ij}\right]dx^i dx^j\right\}$$

$$ds_g^2 = a^2\left\{-(1+2\phi_2)d\eta^2 + 2B_{2,i}d\eta dx^i + \left[(1-2\psi_2)\delta_{ij} + 2E_{2,ij}\right]dx^i dx^j\right\}$$

where ϕ_1, ϕ_2, ψ_1, ψ_2, E_1,E_2, B_1 and B_2 are perturbation fields.

$+\boxed{\chi}$: perturbation of matter field φ

From background EOM, we get $\boxed{z = \dfrac{1+\kappa\rho_0}{1-\kappa p_0}.}$ **: plays key role**

Action in 2nd order

(Lagos, Banados, Ferreira, Garcia-Saenz 2014)

$$S_1[\phi_1, B_1, \psi_1, E_1] = \frac{1}{2}\int d^4x \left\{ \frac{b^2}{\sqrt{z}} \left[4zh\psi_1' E_{1,ii} - 6z\psi_1'^2 - 12zh(\phi_1+\psi_1)\psi_1' - 2\psi_{1,i}(2\phi_{1,i} - \psi_{1,i}) \right.\right.$$
$$- 4h\psi_{1,i}B_{1,i} + 6zh^2(\phi_1+\psi_1)E_{1,ii} - 4\sqrt{z}h(\phi_1+\psi_1)(B_1 - \sqrt{z}E_1')_{,ii}$$
$$- 4\sqrt{z}\psi_1'(B_1 - \sqrt{z}E_1')_{,ii} - 4\sqrt{z}hE_{1,ii}(B_1 - \sqrt{z}E_1')_{,jj} + 4\sqrt{z}hE_{1,ii}B_{1,jj}$$
$$\left. + 3zh^2E_{1,ii}E_{1,jj} + 3zh^2B_{1,i}B_{1,i} - 9zh^2(\phi_1+\psi_1)^2 \right]$$
$$\left. - \frac{2b^4}{\kappa\sqrt{z}} \left[\frac{3}{2}\psi_1^2 - 3\phi_1\psi_1 + \frac{1}{2}B_{1,i}B_{1,i} - \frac{1}{2}E_{1,ii}E_{1,jj} - \frac{1}{2}\phi_1^2 + E_{1,ii}(\phi_1 - \psi_1) \right] \right\},$$

$$S_2[\phi_k, B_k, \psi_k, E_k] = \frac{1}{2}\int d^4x \left\{ \frac{a^2b^2}{\kappa\sqrt{z}} \left[2\sqrt{z}B_{1,i}B_{2,i} + \phi_1\left[(z-1)(3\psi_1 - E_{1,ii}) - 6\psi_2 + 2E_{2,ii} - 2z\phi_2\right] \right.\right.$$
$$+ \psi_1\left[6\psi_2 - (z-1)E_{1,ii} - 2E_{2,ii} - 6z\phi_2\right] - \frac{1}{2}(z-1)(E_{1,ii}E_{1,jj} + B_{1,i}B_{1,i})$$
$$\left. + \frac{3}{2}\left(\phi_1^2 + \psi_1^2\right)(z-1) - 2E_{1,ii}\left(\psi_2 - z\phi_2 + E_{2,ii}\right) \right]$$
$$\left. - \frac{2a^4}{\kappa} \left[\frac{3}{2}\psi_2^2 - \frac{1}{2}\phi_2^2 + \frac{1}{2}B_{2,i}B_{2,i} - \frac{1}{2}E_{2,ii}E_{2,jj} + (\phi_2 - \psi_2)E_{2,ii} - 3\phi_2\psi_2 \right] \right\},$$

$$S_3[\phi_2, B_2, \psi_2, E_2, \chi] = \frac{1}{2}\int d^4x\, a^2 \left\{ \phi_0'^2\left(4\phi_2^2 - B_{2,i}B_{2,i}\right) \right.$$
$$+ \left(\phi_0'^2 - 2V_0a^2\right)\left[\frac{1}{2}\left(3\psi_2^2 - \phi_2^2 + B_{2,i}B_{2,i} - E_{2,ii}E_{2,ii}\right) - 3\phi_2\psi_2 + (\phi_2 - \psi_2)E_{2,ii} \right]$$
$$\left. - 2\phi_0'\chi_{,i}B_{2,i} - 4\phi_0'\chi'\phi_2 + \chi'^2 + 2\left(\phi_2 - 3\psi_2 + E_{2,ii}\right)\left(\chi'\phi_0' - V_1a^2 - \phi_2\phi_0'^2\right) - \chi_{,i}\chi_{,i} - 2V_2a^2 \right\}$$

Gauge (Lagos, Banados, Ferreira, Garcia-Saenz 2014)

Freedom to set to 0 : $(\psi_1, \psi_2, \chi) + (E_1, E_2)$

Our Gauge Choice

$(\cancel{\psi_1}, \psi_2, \chi) + (\cancel{E_1}, E_2)$

→ Matter Field Action containing only background fields:

Matter Action

$$\boxed{S_s[\chi] = \frac{1}{2}\int d^3k d\eta \left[f_1(\eta,k)\chi'^2 - f_2(\eta,k)\chi^2\right],}$$

in Fourier space

$$f_1(\eta,k) = a^2 + \frac{2a^2(z-1)^2\mathcal{X}^2\left[a^2(z-3)-6\kappa h^2 z\right]}{\kappa\sqrt{z}(z+1)(3z-1)},$$

$$f_2(\eta,k) = \frac{\beta}{8\kappa^3 h^2 z^{5/2}(z+1)^2}.$$

$$\beta = a^2\left[\frac{\beta_1}{3z-1}+\frac{\beta_2}{(3z-1)^2}\right],$$

$$\boxed{\mathcal{X} \equiv \frac{1}{a\sqrt{\rho_0+p_0}}, \qquad \mathcal{Y} \equiv -m\frac{\sqrt{\rho_0-p_0}}{\rho_0+p_0}.}$$

$$\begin{aligned}\beta_1 = (z+1)\Big\{&8\kappa^3h^2z^2(3z-1)\Big[k^2\sqrt{z}-12h^2\mathcal{Y}^2z+k^2z^{3/2}+24h^2\mathcal{Y}^2z^2-12h^2\mathcal{Y}^2z^3-3k^2h^2\mathcal{X}^2(z-1)^2(z+1)\\
&+a^6\mathcal{X}^2(z-3)(z-1)^3(3z^2-2z+3)+4\kappa a^4h\mathcal{X}z(z-1)^2\Big[\mathcal{Y}(z-3)^2(3z-1)-3h\mathcal{X}z(3z^2-6z-1)\Big]\\
&+4\kappa^2a^2h^2z(3z-1)\Big[-6h\mathcal{X}\mathcal{Y}(z-3)(z-1)^2z+\mathcal{X}^2(z-1)^2(z+1)[(k^2+9h^2)z-3k^2]\\
&+4\mathcal{Y}^2z(z-3)(z-1)^2+2\kappa m^2z^{3/2}(z+1)\Big]\Big\},\end{aligned}$$

$$\begin{aligned}\beta_2 = (z-1)\Big[a^2(z-3)-6\kappa h^2z\Big]\Big\{&a^4\mathcal{X}^2(z-1)^2(z+1)(3z-1)(3z^2-2z+3)\\
&+4\kappa^2h^2z(3z-1)^2\Big[2z(z-1)(z+1)[2(h+\mathcal{H})\mathcal{X}\mathcal{Y}+(\mathcal{X}\mathcal{Y})']+\mathcal{X}\mathcal{Y}(z^2+6z+1)z'\Big]\\
&+2\kappa a^2h\mathcal{X}\Big[z(z-1)(z+1)(3z-1)(3z^2-2z+3)[(h+4\mathcal{H})\mathcal{X}+2\mathcal{X}']\\
&+\mathcal{X}(9z^5+21z^4-34z^3+30z^2+9z-3)z'\Big]\Big\}.\end{aligned}$$

Canonical Field

$$\boxed{Q = \omega\chi \qquad d\tau = (\omega^2/f_1)d\eta.}$$

Matter Field Eq. becomes

$$\boxed{\ddot{Q} + \left(\sigma_s^2 k^2 - \frac{\ddot{\omega}}{\omega}\right) Q = 0,} \qquad \sigma_s^2 k^2 = \frac{f_1 f_2}{\omega^4}$$

Vacuum **: assume Bunch-Davies vacuum for short wave-length mode**

$$\lim_{k\to\infty} \sigma_s^2 \to 1$$

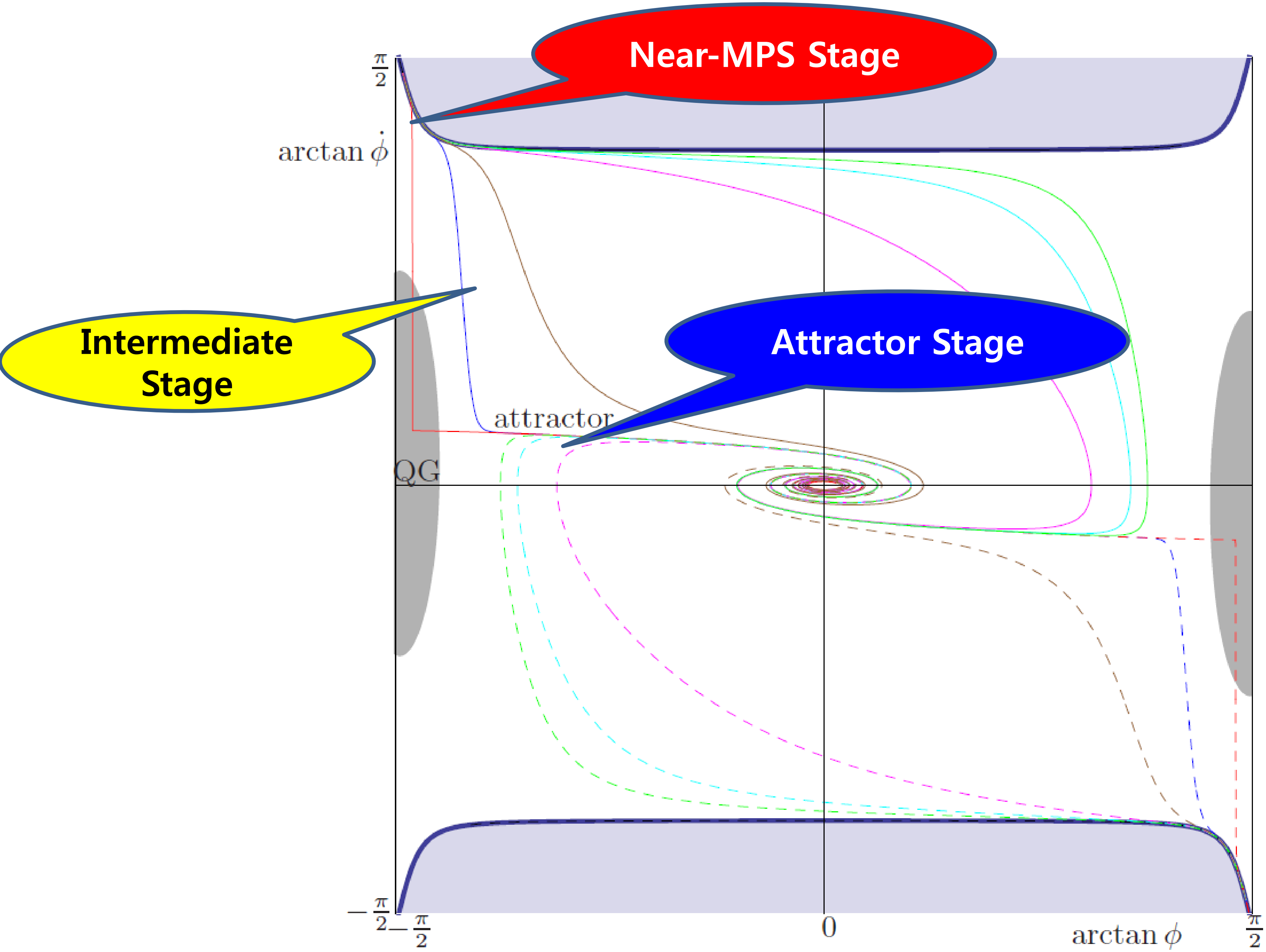
Then, ω is determined as

$$\omega^4 = \frac{a^4}{2\kappa^2 z^2(z+1)(3z-1)} \left\{ a^2\mathcal{X}^2(z-3)(z-1)^2 - 2\kappa z\left[3h^2\mathcal{X}^2(z-1)^2 - \sqrt{z}\right]\right\}$$
$$\times \left\{2a^2\mathcal{X}^2(z-3)(z-1)^2 - \kappa\sqrt{z}\left[12h^2\mathcal{X}^2\sqrt{z}(z-1)^2 - 3z^2 - 2z + 1\right]\right\}.$$

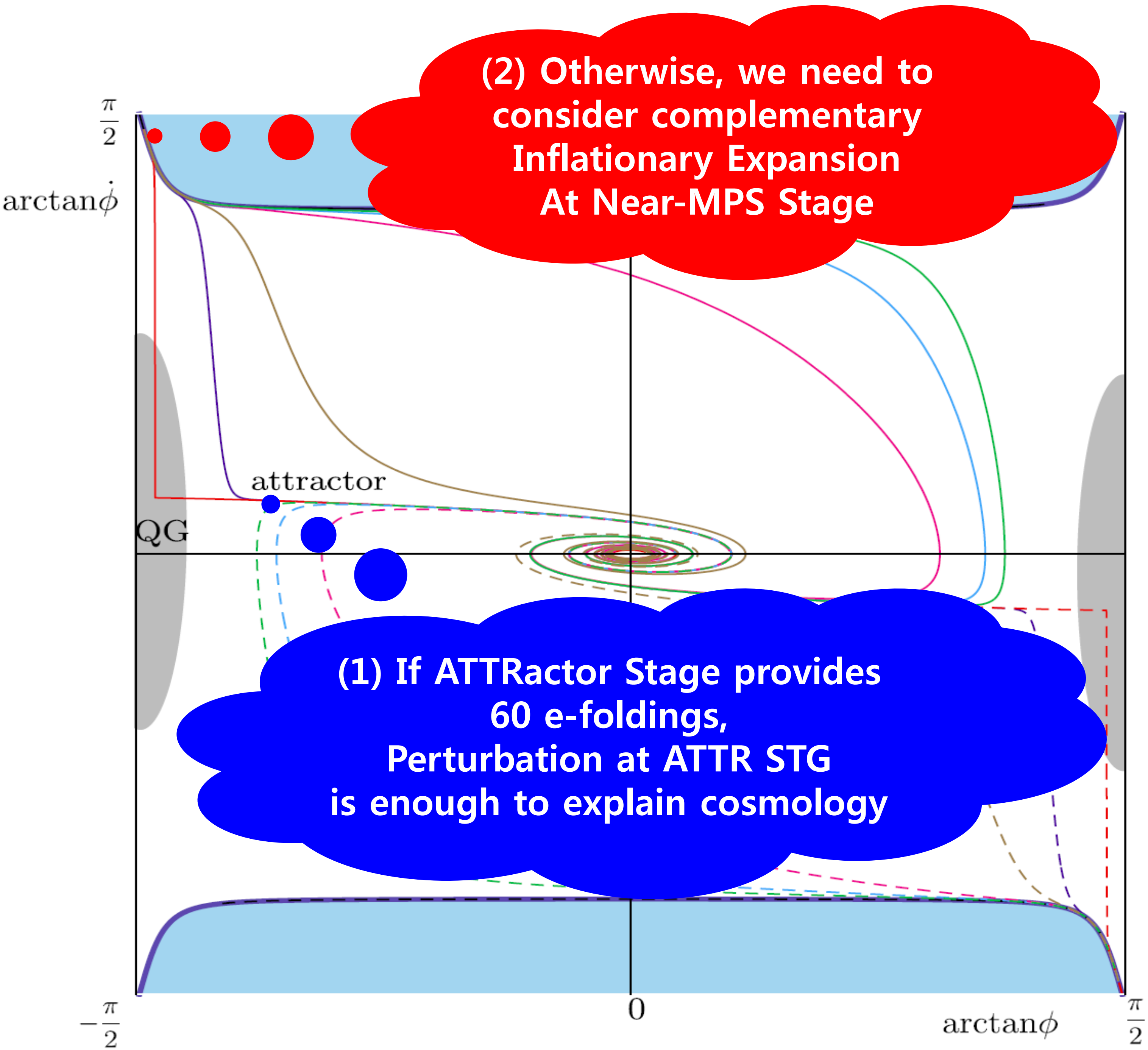
Using EOM1 and EOM2, we get simply

$$\boxed{\omega^4(a,z) = \frac{a^4(3z^2-2z+3)}{z(z+1)(3z-1)}.}$$

$$\boxed{f_1(a,z) = \frac{a^2(3z^2-2z+3)}{(z+1)(3z-1)},}$$



Near-MPS Stage
Intermediate Stage
Attractor Stage
attractor
QG
$\frac{\pi}{2}$
$\arctan \dot{\phi}$
$-\frac{\pi}{2}$
$-\frac{\pi}{2}$
0
$\arctan \phi$
$\frac{\pi}{2}$


(2) Otherwise, we need to consider complementary Inflationary Expansion At Near-MPS Stage
(1) If ATTRactor Stage provides 60 e-foldings, Perturbation at ATTR STG is enough to explain cosmology
attractor
QG
$\frac{\pi}{2}$
$\arctan\dot{\phi}$
$-\frac{\pi}{2}$
0
$\arctan\phi$
$\frac{\pi}{2}$

1) Attractor Stage

$$\boxed{\varphi_0(t) \approx \varphi_i + \sqrt{\frac{2}{3}}mt, \quad a(t) \approx a_i e^{-\varphi_i mt/\sqrt{6}}.}$$

EiBI Correction

$$z = \frac{1+\kappa\rho_0}{1-\kappa p_0} \approx 1 + \boxed{\frac{2}{3}\kappa m^2}$$

: Consider the limit of $\kappa m^2 \ll 1.$

Resulting Approximations

$$\omega^4 \approx a^4\left(1-\frac{4}{3}\kappa m^2\right)$$

$$f_1 \approx a^2\left(1-\frac{2}{3}\kappa m^2\right)$$

$$f_2 \approx a^2\left\{k^2\left(1-\frac{2}{3}\kappa m^2\right) - m^2a^2\left[1+2\kappa m^2(\varphi_i^2-1)\right]\right\}$$

$$d\tau = (\omega^2/f_1)d\eta \approx d\eta$$

EOM

$$\ddot{Q}+\left(\sigma_s^2k^2-\frac{\ddot{\omega}}{\omega}\right)Q\approx\ddot{Q}+\left\{k^2-\frac{\varphi_i^2m^2a^2}{3}\left[1+\frac{3}{\varphi_i^2}+6\kappa m^2\left(1-\frac{4}{3\varphi_i^2}\right)\right]\right\}Q$$

$$\approx\boxed{\ddot{Q}+\left[k^2-\frac{2}{(\tau-\tau_0)^2}\right]Q\approx 0}$$

Solution

$$\boxed{Q\approx\frac{e^{-ik(\tau-\tau_0)}}{\sqrt{2k}}\left[1-\frac{i}{k(\tau-\tau_0)}\right]}$$

2) Near-MPS Stage

MPS Background

$$\varphi_0(t) = \sqrt{\frac{2}{\kappa m^2}}\sinh(mt), \quad a(t) = a_0\left(\frac{\kappa}{2}\right)^{1/3}\cosh^{-2/3}(mt),$$

Near-MPS Background : globally perturbed solution

$$\hat{\varphi}_0 = U\left[1 + \boxed{\psi(t)}\right],$$
$$H = \frac{\hat{a}}{a} = -\frac{2}{3}\frac{dU}{d\varphi_0}\left[1 + \boxed{\gamma(t)}\right],$$

$$\psi = \psi_0 U^{-4/3} e^{t/t_c},$$
$$\gamma = \psi_0\left(-\frac{2}{3} + \sqrt{\frac{2}{3\kappa}}\frac{d\varphi_0}{dU}\right)U^{-4/3}e^{t/t_c},$$

$$U \equiv \sqrt{2[1/\kappa + V(\varphi_0)]}$$
$$t_c = \sqrt{3\kappa/8}.$$

Near-MPS Approximation : z>>1

$$z = \frac{1+\kappa\rho_0}{1-\kappa p_0} = -\frac{1}{\psi}\left(\frac{1+\psi+\psi^2/2}{1+\psi/2}\right) \gg 1$$

Resulting Approximations

$$f_2(a,z) \approx a^2\left(\frac{k^2}{z} + m^2 a^2\right)$$

$$d\tau = \frac{\omega^2}{f_1}d\eta = \frac{\omega^2}{a f_1}dt \approx \frac{dt}{a\sqrt{z}} \approx \frac{\sqrt{-\psi_0}}{a_0}e^{\sqrt{2/3\kappa}t}dt \quad \Rightarrow \quad \tau \approx \sqrt{-\frac{3\kappa\psi_0}{2a_0^2}e^{\sqrt{2/3\kappa}t}}$$

EOM

$$\sigma_s^2 k^2 \approx k^2 + m^2 a^2 z \approx k^2 + \cancel{\frac{3\kappa m^2}{2\tau^2}}, \qquad \frac{\ddot{\omega}}{\omega} \approx \left(-\frac{1}{4} + \cancel{\frac{3}{2}\kappa m^2}\right)\frac{1}{\tau^2}.$$

$$\ddot{Q} + \left(\sigma_s^2 k^2 - \frac{\ddot{\omega}}{\omega}\right)Q \approx \boxed{\ddot{Q} + \left(k^2 + \frac{1}{4\tau^2}\right)Q \approx 0.}$$

Solution

$$\boxed{Q(\tau) = \sqrt{\tau}\Big[c_1 J_0(k\tau) + c_2 Y_0(k\tau)\Big].}$$

Perturbation Production

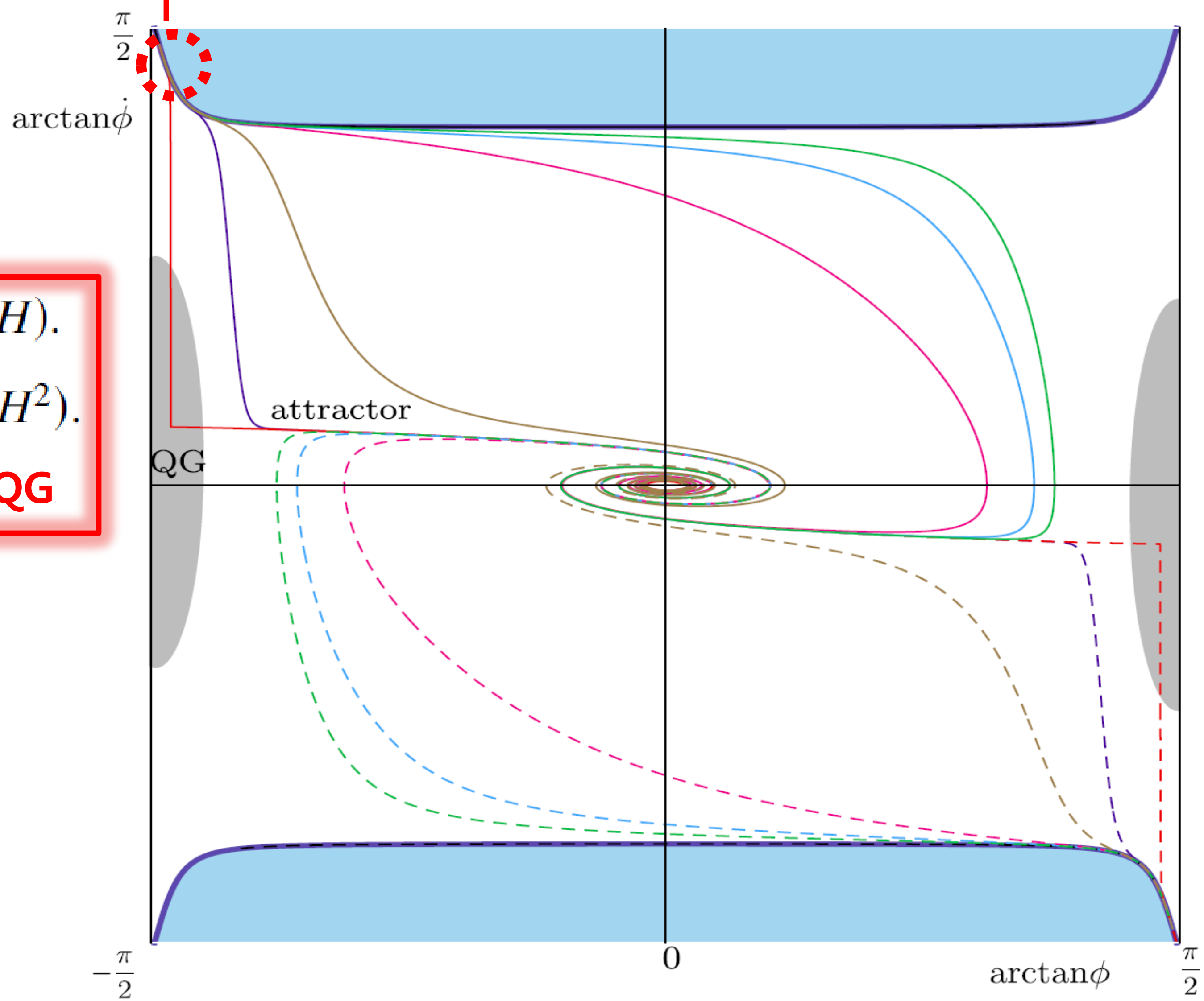
Produced at τ_* due to quantum fluctuation of ϕ

when $\lambda_{\rm phys} = \frac{a(\tau_*)}{k} \gtrsim l_p \Rightarrow \tau_* \gtrsim a^{-1}(kl_p) \approx \sqrt{-\frac{3\kappa\psi_0}{2a_0^2}} \left[\frac{kl_p}{a_0(2\kappa)^{1/3}}\right]^{\sqrt{3/2\kappa m^2}}$

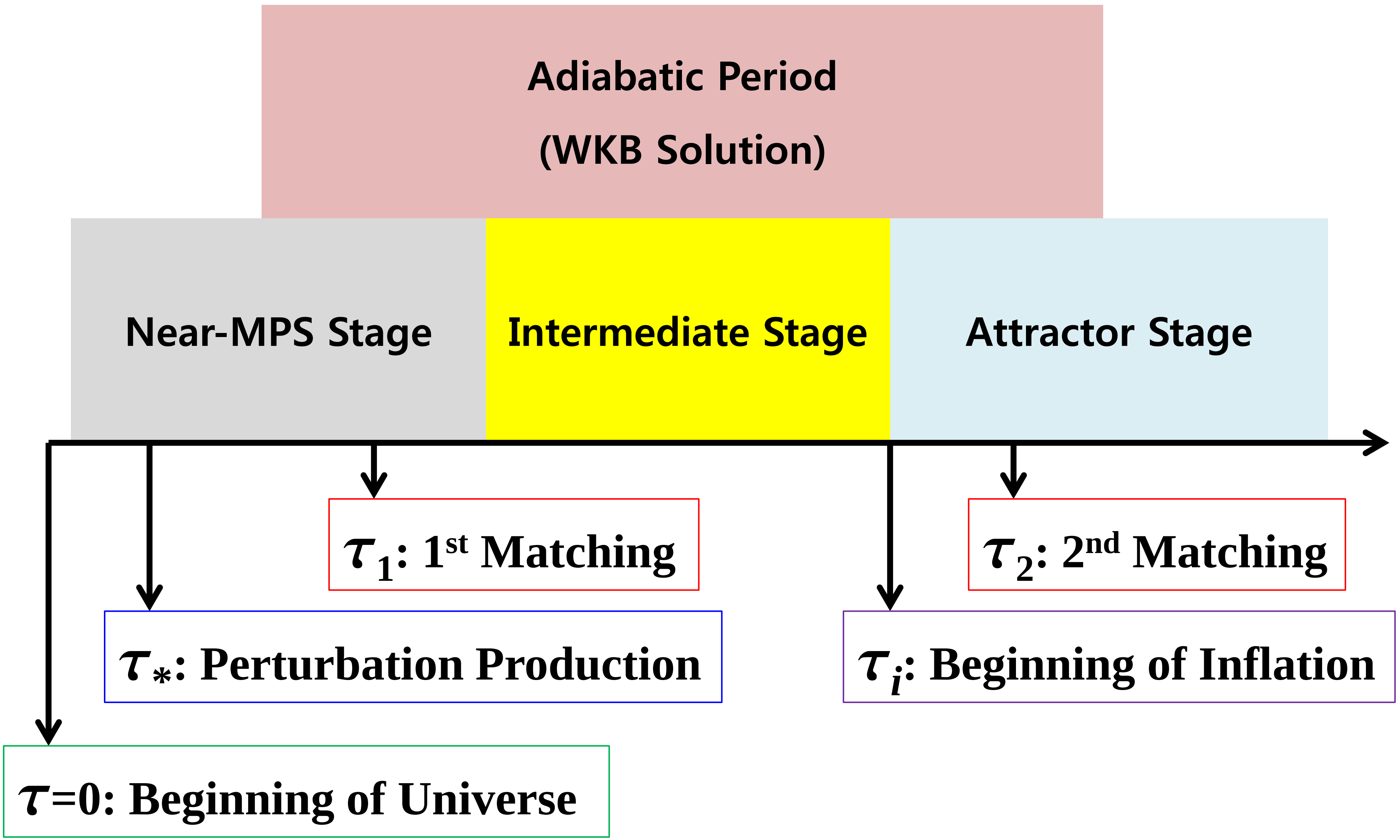
$\delta\phi \sim \mathcal{O}(H).$

$\delta\dot\phi \sim \mathcal{O}(H^2).$

: large at QG



Solution Matching



1) Initial Perturbation produced at Near-MPS Stage

$$\boxed{Q_{\rm MPS}(\tau) = \sqrt{\tau}\left[c_1 J_0(k\tau) + c_2 Y_0(k\tau)\right],}$$

$$c_1 = c_1^{\rm Re} + i c_1^{\rm Im} \equiv \boxed{c,} \qquad c_2 = c_2^{\rm Re} + i c_2^{\rm Im} \equiv \boxed{R - i\frac{\pi}{4c},}$$

Initial Condition : Minimizing Energy

$$\boxed{c^2} = \frac{\pi}{4}\frac{Y^2 + Y_0^2}{|JY_0 - J_0 Y|}, \qquad \boxed{R} = \mp\sqrt{\frac{\pi}{4}}\frac{JY + J_0 Y_0}{\sqrt{|JY_0 - J_0Y|(Y^2 + Y_0^2)}},$$

where $J \equiv (J_0 - 2k\tau_* J_1)/\sqrt{1 + 4k^2\tau_*^2}$, $Y \equiv (Y_0 - 2k\tau_* Y_1)/\sqrt{1 + 4k^2\tau_*^2}$,

$J_{0,1} \equiv J_{0,1}(k\tau_*)$, and $Y_{0,1} \equiv Y_{0,1}(k\tau_*)$.

2) WKB Solution in Adiabatic Period

$$\boxed{\ddot{Q} + \Omega_k^2(\tau) Q = 0}$$

$$\boxed{Q_{\rm WKB}(\tau) = \frac{b_1}{\sqrt{2\Omega_k(\tau)}}\exp\left[i\int^{\tau}\Omega_k(\tau')d\tau'\right] + \frac{b_2}{\sqrt{2\Omega_k(\tau)}}\exp\left[-i\int^{\tau}\Omega_k(\tau')d\tau'\right]}$$

3) Solution at Attractor Stage

$$Q_{\rm ATT}(\tau) = A_1\left[\cos k(\tau-\tau_0) - \frac{\sin k(\tau-\tau_0)}{k(\tau-\tau_0)}\right] + A_2\left[\sin k(\tau-\tau_0) + \frac{\cos k(\tau-\tau_0)}{k(\tau-\tau_0)}\right]$$
$$= A_1'\left[1+\frac{i}{k(\tau-\tau_0)}\right]e^{ik(\tau-\tau_0)} + A_2'\left[1-\frac{i}{k(\tau-\tau_0)}\right]e^{-ik(\tau-\tau_0)},$$

Solution Matching → Determine Coefficients

$$b_{1,2} \approx \frac{c_1 \mp ic_2}{\sqrt{\pi}}\, e^{\pm i(k\tau_1-\pi/4)},$$
$$A'_{1,2} \approx \frac{e^{\mp ik(\tau_2-\tau_0)}}{2}\left[Q_{\rm WKB}(\tau_2;b_1,b_2) \mp \frac{i}{k}\dot{Q}_{\rm WKB}(\tau_2;b_1,b_2)\right].$$

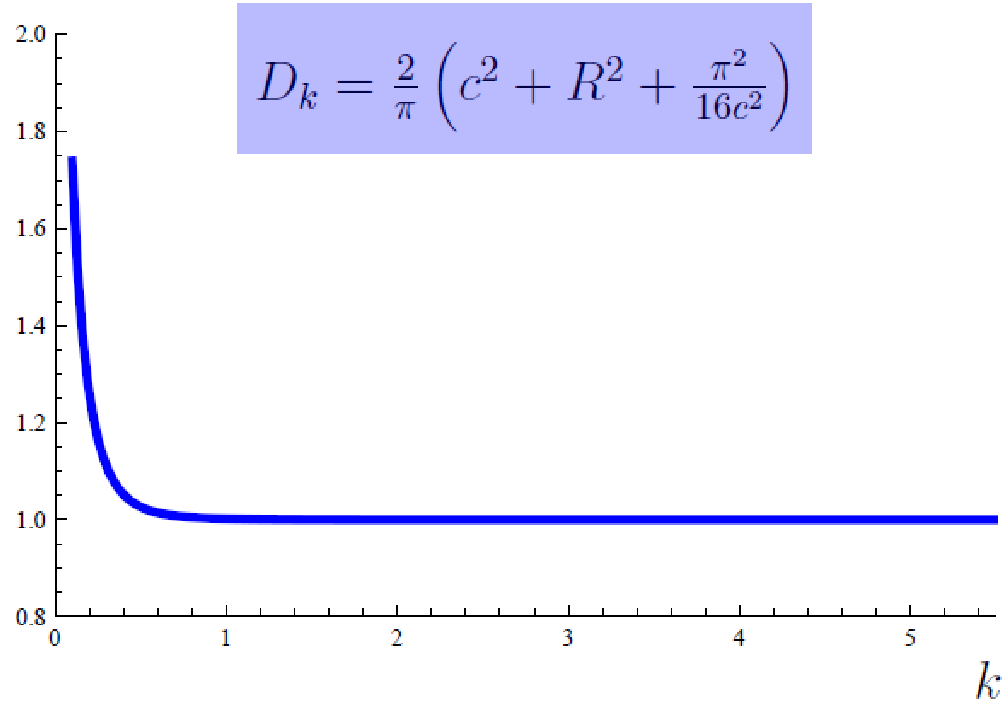
⇨

$$|Q_{\rm ATT}|^2 \approx \frac{|A_1'-A_2'|^2}{k^2(\tau-\tau_0)^2} = \frac{c^2+R^2+\pi^2/16c^2}{\pi k^3(\tau-\tau_0)^2}.$$

Power Spectrum

$$\mathcal{R} = \psi_2 + \frac{H}{\hat{\varphi}_0}\chi_{\text{ATT}} \approx -\frac{1-\kappa m^2}{2}\varphi_i \chi_{\text{ATT}}$$

$$\begin{aligned} P_{\mathcal{R}} &= \frac{k^3}{2\pi^2}\mathcal{R}^2 \approx \frac{k^3}{8\pi^2}(1-\kappa m^2)^2 \varphi_i^2 \left|\frac{Q_{\text{ATT}}}{\omega_{\text{ATT}}}\right|^2 \\ &\approx \frac{2}{\pi}\left(c^2 + R^2 + \frac{\pi^2}{16c^2}\right) \times \frac{(1-\kappa m^2)^2}{(1-4\kappa m^2/3)^{1/2}} \times \frac{m^2\varphi_i^4}{96\pi^2} \\ &\equiv D_k \times E_\kappa^{\text{S}} \times P_{\mathcal{R}}^{\text{GR}} \end{aligned}$$



$$D_k = \frac{2}{\pi}\left(c^2 + R^2 + \frac{\pi^2}{16c^2}\right)$$

$a_0 = -\psi_0 = 0.1$, $\lambda = 1$, $m = 10^{-5}$, $\kappa = 10^6$, and $\tau_* = 1$.

Tensor Perturbation

Formulation of Tensor Perturbation in EiBI

$$g_{\mu\nu}dx^\mu dx^\nu = -\boxed{a^2}d\eta^2 + a^2\left(\delta_{ij} + h_{ij}\right)dx^i dx^j\,,$$

$$q_{\mu\nu}dx^\mu dx^\nu = -X^2 d\eta^2 + Y^2\left(\delta_{ij} + \gamma_{ij}\right)dx^i dx^j\,,$$

$$= \boxed{Y^2}\left[-d\tau^2 + \left(\delta_{ij} + \gamma_{ij}\right)dx^i dx^j\right]\,,$$

a, Y : scale factors

$$\boxed{\partial_i h^{ij} = \partial_i \gamma^{ij} = 0 \text{ and}} \quad \boxed{h = \gamma = 0.}$$

From EOM 1, one gets $\boxed{h_{ij} = \gamma_{ij}.}$

From EOM 2,

$$\frac{\kappa Y^2}{2X^2}h''_\lambda + \frac{\kappa Y^2}{2X^2}\left(3\frac{Y'}{Y} - \frac{X'}{X}\right)h'_\lambda + \frac{\kappa k^2}{2}h_\lambda = 0.$$

Canonical Field

$$\boxed{h_\lambda = f_0 \frac{\mu_\lambda}{Y}. \qquad d\tau = (X/Y)d\eta = (X/aY)dt,}$$

$$\boxed{\ddot{\mu}_\lambda + \left(k^2 - \boxed{\frac{\ddot{Y}}{Y}}\right)\mu_\lambda = 0}$$

: **Perturbation is governed by Y, NOT by a**

1) Attractor Stage

Same with Ordinary Inflation

$$\boxed{Y} = \kappa^{1/2}(\bar{\lambda} - p)^{1/4}(\bar{\lambda} + \rho)^{1/4} a$$

$$\approx \kappa^{1/2}\sqrt{\bar{\lambda} + \frac{1}{2}m^2\phi_i^2 + 2m^2 \log\left(\frac{\tau - \tau_i}{\tau_i - \tau_0} + 1\right)}\, a \equiv \boxed{Y_0 a.}$$

Solution

$$\boxed{\mu_\lambda(\tau) = A_1\left\{\left[\cos k(\tau - \tau_0) - \frac{\sin k(\tau - \tau_0)}{k(\tau - \tau_0)}\right] + i\left[\sin k(\tau - \tau_0) + \frac{\cos k(\tau - \tau_0)}{k(\tau - \tau_0)}\right]\right\}}$$

2) Near-MPS Stage (Background: MPS sol + Global Perturbation)

Scale Factors

$$a(\tau) \approx \frac{3\tau_m}{2m}\tau^{m\sqrt{2\kappa/3}},$$

$$\boxed{Y} = \kappa^{1/2}(\lambda - p)^{1/4}(\lambda + \rho)^{1/4} a \approx \frac{\kappa^{1/4} a_0^{3/2}}{\sqrt{2t_c}}\sqrt{\tau} \equiv \boxed{\tau_Y\sqrt{\tau},}$$

$$Y \propto a$$

Solution

$$\mu_\lambda(\tau) = \sqrt{\tau}[c_1 J_0(k\tau) + c_2 Y_0(k\tau)].$$

Solution Matching :

Exactly the SAME
with Scalar Perturbation

Power Spectrum

$$
\begin{aligned}
P_{\mathrm{T}} &= \frac{2k^3}{\pi^2}\left|\frac{\mu_{\mathrm{ATT}}}{Y}\right|^2 \\
&\approx \frac{2}{\pi}\left(c^2 + R^2 + \frac{\pi^2}{16c^2}\right) \times \frac{1}{1+\kappa m^2\varphi_i^2/2} \times \frac{m^2\varphi_i^2}{6\pi^2} \\
&\equiv D_k \times E_\kappa^{\mathrm{T}} \times P_{\mathrm{T}}^{\mathrm{GR}}
\end{aligned}
$$

Tensor-to-Scalar Ratio

$$P_{\mathrm{T}} = \frac{2k^3}{\pi^2}\left|\frac{\mu_{\mathrm{ATT}}}{Y}\right|^2$$

$$\approx \frac{2}{\pi}\left(c^2 + R^2 + \frac{\pi^2}{16c^2}\right) \times \frac{1}{1+\kappa m^2\varphi_i^2/2} \times \frac{m^2\varphi_i^2}{6\pi^2}$$

$$\equiv D_k \times E_\kappa^{\mathrm{T}} \times P_{\mathrm{T}}^{\mathrm{GR}}$$

$$P_{\mathcal{R}} = \frac{k^3}{2\pi^2}\mathcal{R}^2 \approx \frac{k^3}{8\pi^2}(1-\kappa m^2)^2\varphi_i^2\left|\frac{Q_{\mathrm{ATT}}}{\omega_{\mathrm{ATT}}}\right|^2$$

$$\approx \frac{2}{\pi}\left(c^2 + R^2 + \frac{\pi^2}{16c^2}\right) \times \frac{(1-\kappa m^2)^2}{(1-4\kappa m^2/3)^{1/2}} \times \frac{m^2\varphi_i^4}{96\pi^2}$$

$$\equiv D_k \times E_\kappa^{\mathrm{S}} \times P_{\mathcal{R}}^{\mathrm{GR}}$$

$$r = \frac{P_{\mathrm{T}}}{P_{\mathcal{R}}} \approx \frac{E_\kappa^{\mathrm{T}} \times P_{\mathrm{T}}^{\mathrm{GR}}}{E_\kappa^{\mathrm{S}} \times P_{\mathcal{R}}^{\mathrm{GR}}} = \frac{(1-4\kappa m^2/3)^{1/2}}{(1-\kappa m^2)^2(1+\kappa m^2\varphi_i^2/2)}\, r^{\mathrm{GR}} \approx \boxed{\frac{1+4\kappa m^2/3}{1+\kappa m^2\varphi_i^2/2}}\, r^{\mathrm{GR}}$$

$$r \approx \boxed{\frac{1+4\kappa m^2/3}{1+\kappa m^2\varphi_i^2/2}}\, r^{\mathrm{GR}}$$

$$\boxed{r^{\mathrm{GR}} \sim 0.131 \text{ for } 60\ e\text{-foldings.}}$$

$$\boxed{\kappa m^2 \lesssim \mathcal{O}(10^{-2})}$$

$$\boxed{\kappa m^2 \varphi_i^2 \sim \mathcal{O}(1)} \quad \Leftarrow \quad \varphi_i \sim \mathcal{O}(10)$$

Therefore, r can be significantly lowered.

PLANCK predicts r <0.09.

Conclusions

1. EiBI gravity provides

   Non-Singular, Non-Quantum Gravitational, Natural pre-Inflationary Stage

2. Density Perturbation :

   may leave a peculiar signature in CMB

3. Tenor-to-Scalar ratio :

   can be significantly LOWERED.